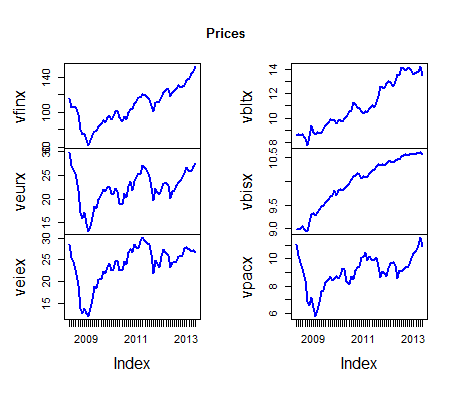
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|  | **Summer 2013** |
|  | Final Project Econ 424  **Garrett Black** |

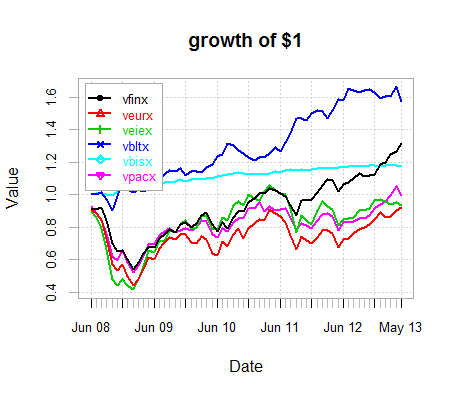
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| **[portfolio Analysis: Vanguard Funds]**  **Executive Summary**   * **Background: The S&P 500 (VFINX), the European Stock Exchange (VEURX), An emerging Markets Fund (VEIEX), Long and Short-Term Bonds (VBLTX and VBISX), and the MSCI Pacific index (VPACX) were analyzed in order to produce risk and return estimates in conjunction with portfolio recommendations.** * **method: through convex combinations of various minimum variance and tangency portfolios, while targeting certain returns, several different efficient portfolios were achieved. estimated returns, along with standard error predictions, were also produced. monthly historical prices and returns from may 2008 to may 2013 were also utilized as comparisons, with 60 data points overall.** * **Results: overall, the safest assets in which to invest were primarily vbisx, with vbltx a distant second. however, to achieve target returns, it was best to short sell veiex, veurx, and vpacx, while investing in vfinx to an extent, and reserving some funds in vbisx for security.** |
|  |

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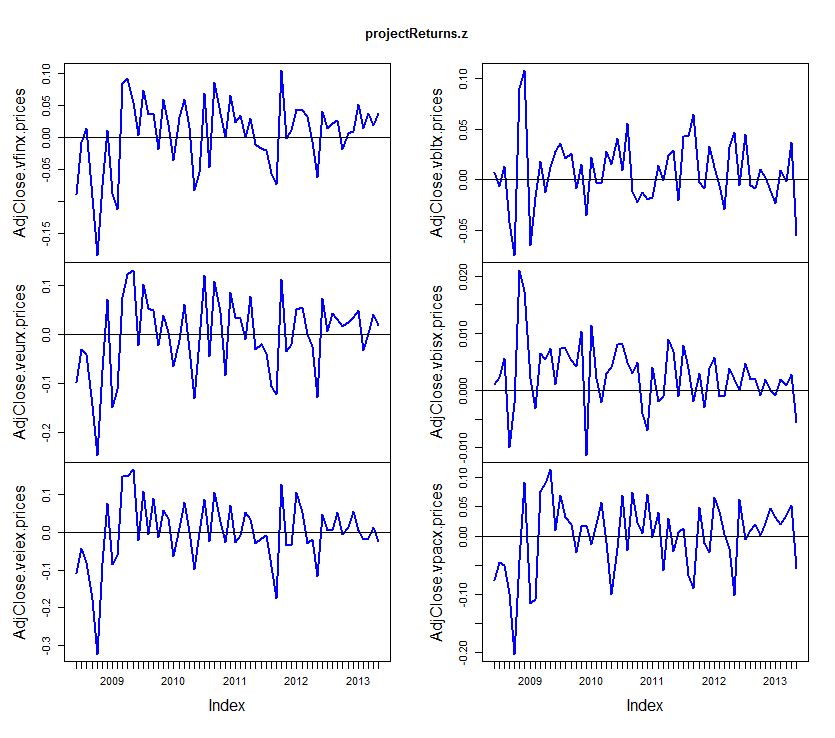
1. Return Calculations and Plots
2. Sample Statistics
3. Value-at-Risk Calculations
4. Rolling CER Parameter Analysis
5. Portfolio Theory
6. Asset Allocation
7. Single Index and CAPM

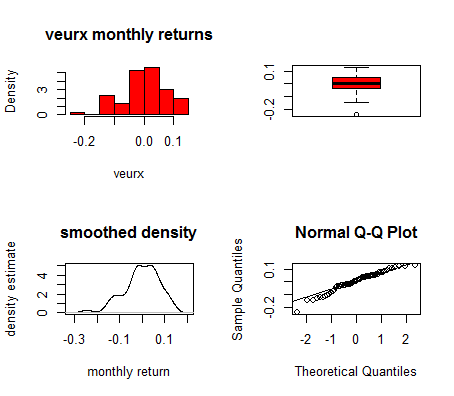
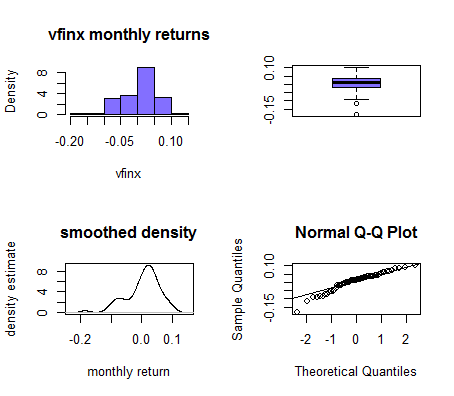
**1. Return Calculations and Plots**

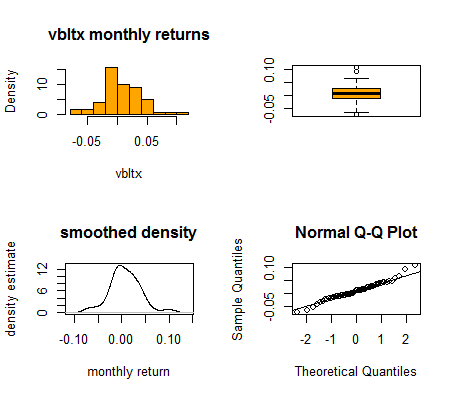
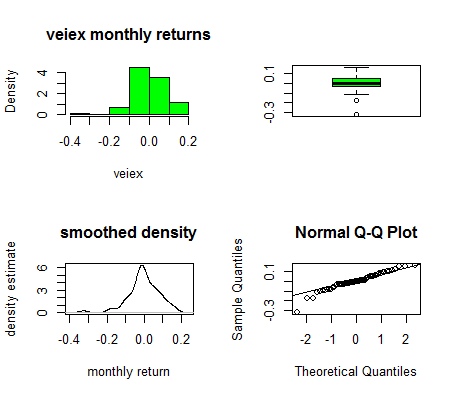


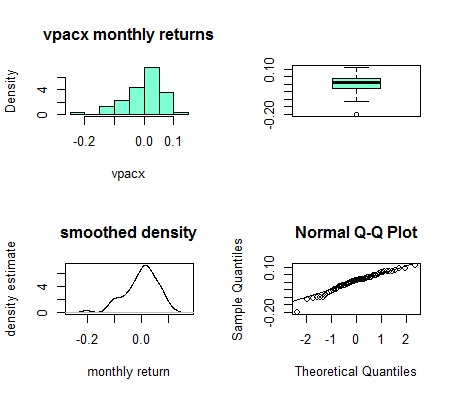
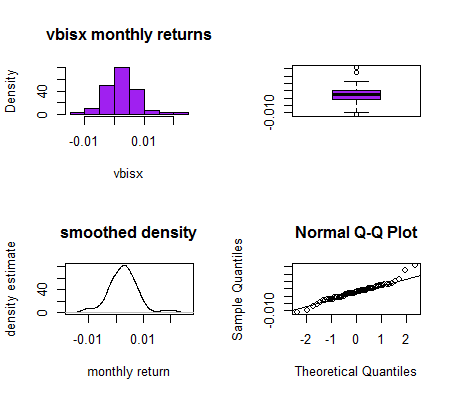


VBLTX gives the highest future value, which isn’t quite surprising, since people would have bought plenty of bonds in the wake of the financial crisis, seeking safer returns. VEURX’s returns seem rather small, along with VEIEX. VFINX’s returns contrast, seeming pretty sizable. The sizable dip in all indices is obviously the 2008 crisis, with corresponding recoveries. The various European PIIGS’ crises probably led to its various dips and bends, with corresponding dips in VEIEX. VPACX would go along with VEIEX, since many emerging economies are tied to China’s vast consumption of raw materials. Whereas the recovery of corporations overall would lead to the VFINX climbing. The bond returns are most likely explained by flaps in the market and expectation of rising interest rates. VBISX and VBLTX have a few outliers on the positive side, probably spiked by the 2008 crisis. All the stocks have negative 2008 outliers as well.









**2. Descriptive/Sample Statistics**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Index** | **Mean** | **St.Dev** | **Excess Kurtosis** | **Sharpe Slopes** | **Skew** | **Variance** | **1% Quantile** | **5% Quantile** |
| **vfinx** | **0.00457** | **0.05550** | **1.00847** | **0.07483** | **-.90382** | **0.00308** | **-0.14185** | **-0.08852** |
| **veurx** | **-0.0014** | **0.07760** | **0.36904** | **-0.0235** | **-.63236** | **0.00602** | **-0.18751** | **-0.13116** |
| **veiex** | **-0.0012** | **0.08504** | **2.34631** | **-0.0191** | **-.83375** | **0.00723** | **-0.23748** | **-0.12043** |
| **vbltx** | **0.00756** | **0.03296** | **1.06735** | **0.21674** | **0.28229** | **0.00109** | **-0.06889** | **-0.04298** |
| **vbisx** | **0.00269** | **0.00549** | **1.88930** | **0.41300** | **0.40662** | **0.00003** | **-0.01051** | **-0.00573** |
| **vpacx** | **-0.0001** | **0.06103** | **0.78072** | **-0.0091** | **-.77352** | **0.00372** | **-0.15220** | **-0.10309** |

VBLTX has the highest average return while VEURX limps behind at the lowest average return (has the greatest negative number, so lowest return). VEIEX has a pretty high standard deviation, the highest as a matter of fact, while VBISX is quite low. VBISX also has the highest Sharpe’s slope, meaning it has the best return-risk ratio. VBLTX looks fairly normally distributed, but VBISX has the lowest variance. Consulting the graphs below, it’s easier to see that VBLTX and VBISX are closest to normal distributions, but still have enough skew and variance they don’t quite fit.

**Standard Errors**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **muhat.vals** | **se.muhat** | **sigmahat** | **se.sigmahat** | **sigma2hat.** | **se.sigma2hat** |
| **vfinx** | 0.00457 | 0.00716 | 0.05550 | 0.00507 | 0.00308 | 0.00056 |
| **veurx** | -0.00141 | 0.01002 | 0.07760 | 0.00708 | 0.00602 | 0.00110 |
| **veiex** | -0.00121 | 0.01098 | 0.08504 | 0.00776 | 0.00723 | 0.00132 |
| **vbltx** | 0.00756 | 0.00426 | 0.03296 | 0.00301 | 0.00109 | 0.00020 |
| **vbisx** | 0.00269 | 0.00071 | 0.00549 | 0.00050 | 0.00003 | 0.00001 |
| **vpacx** | -0.00014 | 0.00788 | 0.06103 | 0.00557 | 0.00372 | 0.00068 |

VBISX far and away has the smallest standard error; the other estimations are decent, except for VEURX and VEIEX. The estimated numbers are more accurate than the actual standard deviations.

**Confidence Intervals**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **vfinx** | **veurx** | **veiex** | **vbltx** | **vbisx** | **vpacx** |
| **mu.lower** | **-0.00976** | **-0.02144** | **-0.02316** | **-0.00095** | **0.00127** | **-0.01589** |
| **mu.upper** | **0.01890** | **0.01863** | **0.02075** | **0.01607** | **0.00410** | **0.01562** |
| **sigma.lower** | **0.04536** | **0.06343** | **0.06951** | **0.02695** | **0.00449** | **0.04988** |
| **sigma.upper** | **0.06563** | **0.09176** | **0.10057** | **0.03898** | **0.00650** | **0.07217** |
| **sigma2.lower** | **0.00196** | **0.00382** | **0.00459** | **0.00069** | **0.00002** | **0.00236** |
| **sigma2.upper** | **0.00420** | **0.00822** | **0.00987** | **0.00148** | **0.00004** | **0.00508** |

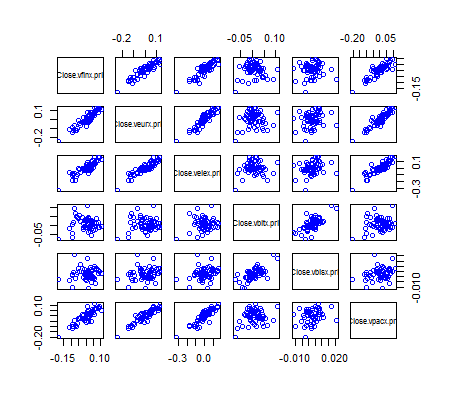
Given that all the indices except VBISX cross 0, there’s a fair amount of uncertainty in all but VBISX. And accordingly, VBISX has the most exact intervals.

**Annual Statistics**

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Annual St.Dev of Return** | **Annual Mean Return** | **$1 Growth in 5 Yrs.** |
| **vfinx** | **0.19225** | **0.05483** | **1.315412099** |
| **veurx** | **0.26881** | **-0.01691** | **0.918925708** |
| **veiex** | **0.29459** | **-0.01449** | **0.930112251** |
| **vbltx** | **0.11419** | **0.09074** | **1.574125689** |
| **vbisx** | **0.01903** | **0.03223** | **1.174861185** |
| **vpacx** | **0.21140** | **-0.00164** | **0.991833528** |

The annual returns reflect some interesting points. Namely, VBLTX appears to be the highest-returning fund, while the S&P 500 is no slouch either. $1 would grow similarly to the calculations above.

**Pairwise Plots**



VFINX, VEURX, VEIEX and VPACX appear fairly related, with the strongest relationship between VFINX and VEURX. This all makes sense; the S&P 500 and European markets would intertwine due to trade concerns, while corporations rely on emerging markets for raw materials, as well as the VPACX producers. Meanwhile, the bond markets would be only slightly related.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Covariance** | | | | | | |
| **Asset** | **vfinx** | **veurx** | **veiex** | **vbltx** | **vbisx** | **vpacx** |
| **vfinx** | 0.00308 | 0.00397 | 0.00411 | 0.00001 | 0.00003 | 0.00297 |
| **veurx** | 0.00397 | 0.00602 | 0.00606 | 0.00023 | 0.00009 | 0.00437 |
| **veiex** | 0.00411 | 0.00606 | 0.00723 | 0.00034 | 0.00011 | 0.00464 |
| **vbltx** | 0.00001 | 0.00023 | 0.00034 | 0.00109 | 0.00012 | 0.00038 |
| **vbisx** | 0.00003 | 0.00009 | 0.00011 | 0.00012 | 0.00003 | 0.00008 |
| **vpacx** | 0.00297 | 0.00437 | 0.00464 | 0.00038 | 0.00008 | 0.00372 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Correlation** | | | | | | |
| **Asset** | **vfinx** | **veurx** | **veiex** | **vbltx** | **vbisx** | **vpacx** |
| **vfinx** | 1.0000 | 0.9227 | 0.8705 | 0.0032 | 0.0885 | 0.8781 |
| **veurx** | 0.9227 | 1.0000 | 0.9190 | 0.0916 | 0.2098 | 0.9235 |
| **veiex** | 0.8705 | 0.9190 | 1.0000 | 0.1226 | 0.2260 | 0.8934 |
| **vbltx** | 0.0032 | 0.0916 | 0.1226 | 1.0000 | 0.6797 | 0.1881 |
| **vbisx** | 0.0885 | 0.2098 | 0.2260 | 0.6797 | 1.0000 | 0.2277 |
| **vpacx** | 0.8781 | 0.9235 | 0.8934 | 0.1881 | 0.2277 | 1.0000 |

As seen above, VFINX, VEURX, VEIEX, and VPACX are strongly correlated, with VFINX and VEURX most strongly correlated. VBLTX is barely correlated with anything apart from VBISX. It appears that VEIEX and VPACX covary together quite well, as do VEURX with VFINX, and VEIEX with VFINX. Overall, moving between asset types appears to produce the lack of correlation.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Asset** | **RhoHats** | **RhoHat SEs** | **Upper Rho - 95%** | **Lower Rho - 95%** |
| VFINX,VEURX | 0.922745 | 0.019177 | 0.961098 | 0.884391 |
| VFINX,VEIEX | 0.870545 | 0.031262 | 0.933068 | 0.808022 |
| VFINX,VBLTX | 0.003250 | 0.129098 | 0.261446 | -0.254946 |
| VFINX,VBISX | 0.088483 | 0.128089 | 0.344660 | -0.167694 |
| VFINX,VPACX | 0.878141 | 0.029547 | 0.937234 | 0.819047 |
| VEURX,VEIEX | 0.919011 | 0.020065 | 0.959140 | 0.878881 |
| VEURX,BLTX | 0.091579 | 0.128017 | 0.347612 | -0.164455 |
| VEURX,VBISX | 0.209822 | 0.123416 | 0.456653 | -0.037010 |
| VEURX,PACX | 0.923510 | 0.018994 | 0.961499 | 0.885521 |
| VEIEX,VBLTX | 0.122612 | 0.127159 | 0.376929 | -0.131705 |
| VEIEX,VBISX | 0.225996 | 0.122506 | 0.471008 | -0.019016 |
| VEIEX,VPACX | 0.893430 | 0.026050 | 0.945530 | 0.841330 |
| VBLTX,VBISX | 0.679717 | 0.069454 | 0.818624 | 0.540810 |
| VBLTX,VPACS | 0.188079 | 0.124533 | 0.437145 | -0.060986 |
| VBISX,VPACX | 0.227703 | 0.122406 | 0.472515 | -0.017108 |

Given the standard errors on the correlation values, the estimates are more precise within an asset type. As for the diversification, since the correlation appears to be fairly weak in many instances, spreading across various assets could improve matters, yet since these portfolios already seem fairly diversified, there may not be much more benefit to reap.

**3. Value-at-Risk Calculations**

**Monthly VaRs**

|  |  |  |
| --- | --- | --- |
| Asset | **VaR.01** | **VaR.05** |
| **vfinx** | **-11709.2756** | **-8306.116711** |
| **veurx** | **-16633.84908** | **-12106.60283** |
| **veiex** | **-18048.50303** | **-13158.67466** |
| **vbltx** | **-6679.129948** | **-4558.915355** |
| **vbisx** | **-1004.297974** | **-633.0020729** |
| **vpacx** | **-13246.79733** | **-9562.790481** |

VBISX has lowest VaR, while VEIEX dominates the risk overall at both 1% and 5%.

**Annual VaRs**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Asset** | **Annual St.Dev** | **Annual Mean Return** | **VaR.05.annual** | **VaR.01.annual** |
| **vfinx** | **0.19225** | **0.05483** | **-23002.18811** | **-32457.31401** |
| **veurx** | **0.26881** | **-0.01691** | **-36812.46932** | **-47389.55995** |
| **veiex** | **0.29459** | **-0.01449** | **-39289.13688** | **-50331.97685** |
| **vbltx** | **0.11419** | **0.09074** | **-9252.192985** | **-16046.37963** |
| **vbisx** | **0.01903** | **0.03223** | **92.8859503** | **-1196.820684** |
| **vpacx** | **0.2114** | **-0.00164** | **-29486.22714** | **-38947.12947** |

VEIEX still is the highest-risk component, while VBISX still resolutely stays low.

**Empirical Value-at-Risk Calculations**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Asset** | **Means** | **St.Dev** | **VaR.05** | **VaR.01** | **VaR.05.Annual** | **VaR.01.Annual** |
| VFINX | 0.0548 | 0.1922 | -81313.5604 | -92852.3208 | -23001.5550 | -32456.5724 |
| VEURX | -0.0169 | 0.2688 | -81215.3343 | -90912.8025 | -36811.7676 | -47388.8354 |
| VEIEX | -0.0145 | 0.2946 | -75388.7104 | -93785.9262 | -39289.1531 | -50332.0305 |
| VBLTX | 0.0907 | 0.1142 | -78416.4822 | -90164.7728 | -9253.0834 | -16047.4159 |
| VBISX | 0.0322 | 0.0190 | -78405.5742 | -90953.2912 | 92.5480 | -1197.0966 |
| VPACX | -0.0016 | 0.2114 | -81493.4837 | -91724.4910 | -29485.9112 | -38946.7830 |

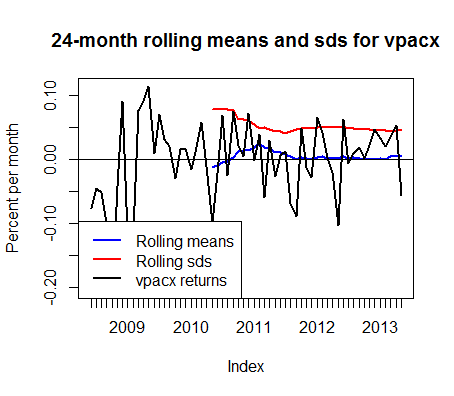
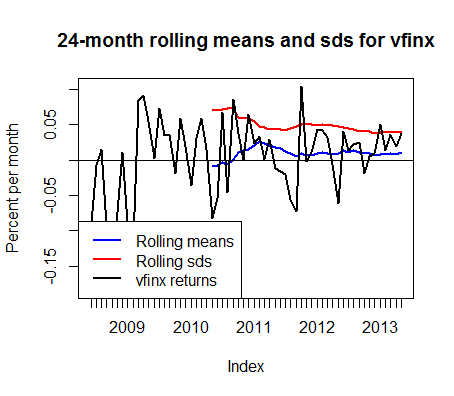
The empirical results equalize the risk between all the stock indices much more; the numbers are much closer together. VBISX, of course, stays quite low. Thus, using the empirical results, we see how the differences in reality are not quite as vast as prior numbers would have us believe.

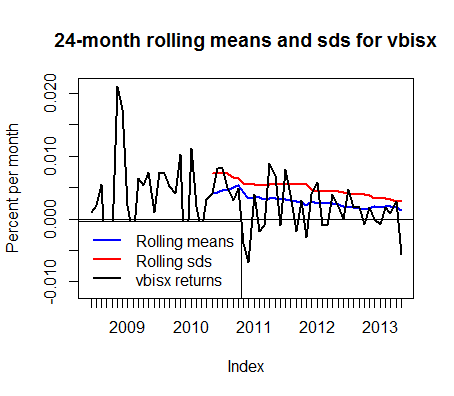
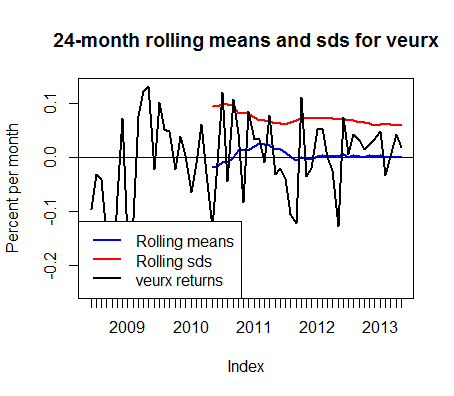
**Boot Statistics**

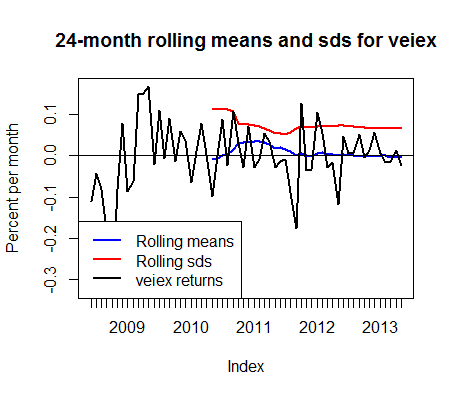
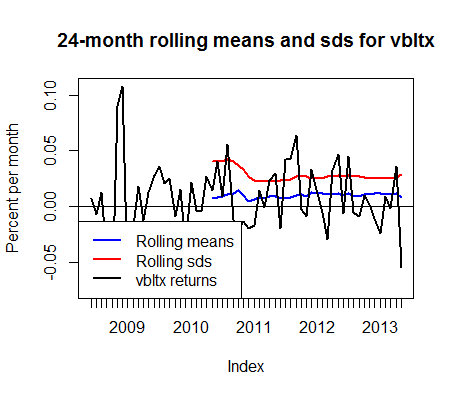
|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | BootStrap |  |  | 95% Level |  |
|  | Original | Bias | Standard Error | Normal | Percentile |
| vfinx | -8306.117 | 195.0595 | 1372.952 | (-11191, -5809) | (-10794, -5396) |
| veurx | -12106.6 | 117.0513 | 1670.458 | (-15498, -8950) | (-15224, -8667) |
| veiex | -13158.67 | 62.939222 | 2198.795 | (-17531, -8912) | (-17550, -8967) |
| vbltx | -4558.915 | 65.85308 | 652.6469 | (-5904, -3346) | (-5757, -3187) |
| vbisx | -633.0021 | 16.56643 | 114.852 | (-874.7, -424.5) | (-845.9, -391.4) |
| vpacx | -9562.79 | 174.6323 | 1434.159 | (-12548, -6927) | (-12157, -6637) |

The standard errors bear out my conclusions somewhat thus far; VBISX still offers the safest bet, whereas the high errors for VEIEX and VEURX may imply the errors are overstated, somewhat. Overall, the estimates are somewhat precise; the errors seem to hover too high for my liking.

## 4. Rolling Analysis of the CER Model Parameters







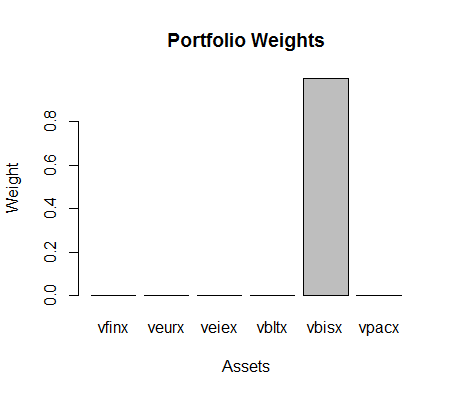
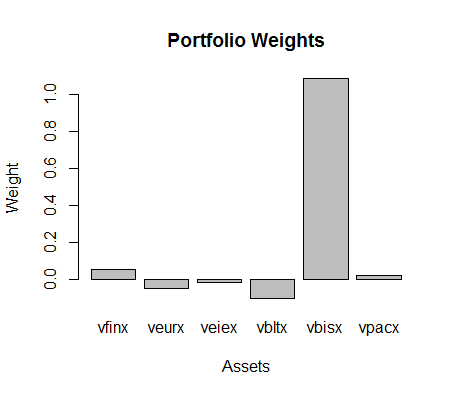
Overall, after some odd jumps or humps in the first half of 2011 (with escalating European crises rippling worldwide), the means and standard deviations stabilize somewhat, mostly. They continue on their slight upward or downward trends, depending on the index. Tables for the numerical values are in an appendix.

## 5. Portfolio Theory

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Portfolio | Expected Return | Standard Deviation | Variance  (E-05) | Annualized Return | Annualized SD | Sharpe’s Ratio |
| Global Minimum | 0.0024980 | 0.0042254 | 1.78544 | 0.0299754 | 0.0146374 | 0.4925583 |
| Global Min. No Shorts | 0.0026874 | 0.0054929 | 3.01723 | 0.0322489 | 0.0190281 | 0.4133927 |
| Efficient (Target Max Ret) | 0.0075614 | 0.0160425 | 25.7363 | 0.0907362 | 0.0555730 | 0.4453568 |
| Tangency | 0.0034162 | 0.0050727 | 2.57321 | 0.0409949 | 0.0175723 | 0.5913124 |
| Tangency  No Shorts | 0.0027028 | 0.0055116 | 3.03773 | 0.0324340 | 0.0190927 | 0.414795 |

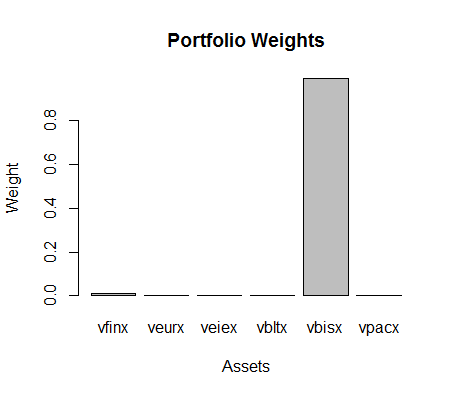
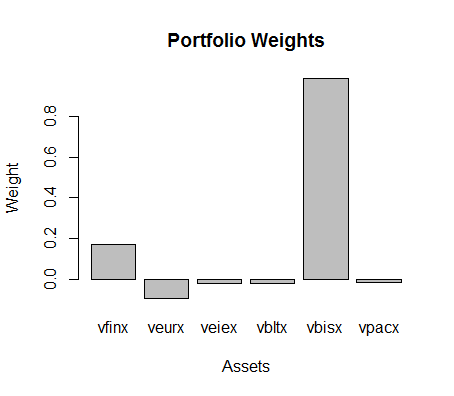
With short sales, expected return went up for tangency and declined for global minimum. Tangency with shorts also improved in Sharpe’s ratio compared to that without shorts. Sharpe’s ratio for each asset under the tangency portfolio showed that VBISX had the highest Sharpe’s ratio as well, if you compare it to that above.

**Global Minimum With Short Sales and Without Weights**



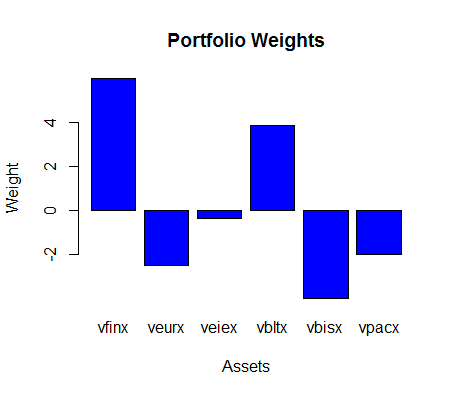
So with short sales, there are some negative weights, meaning that VEURX, VBLTX, and VEIEX will be shorted slightly, in this global minimum variance portfolio. Without the option of short sales, VBISX becomes our safest bet.

**Tangency Portfolio Weights With Short Sales and Without**



Again, some negative weights, so shorting of VEURX primarily combined with shorting of VPACX, VBLTX, and VEIEX.

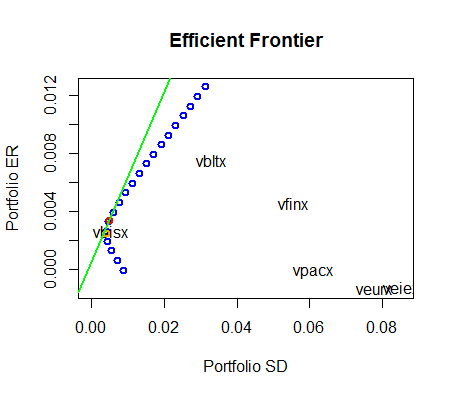
**Efficient Portfolio Max**



**Value-at-Risk Calculations**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | VaR.01.simple | VaR.05.simple | GminPort .01 | GminPort.05 | GminPortNS.01 | GminPortNS.05 |
| vfinx | -11374.48 | -8019.79 | -640.38 | -451.51 | -11.37 | -8.02 |
| veurx | -16071.29 | -11612.23 | 750.53 | 542.29 | 0.00 | 0.00 |
| veiex | -17316.60 | -12521.89 | 264.94 | 191.58 | 0.00 | 0.00 |
| vbltx | -6721.79 | -4573.87 | 666.80 | 453.73 | 0.00 | 0.00 |
| vbisx | -1007.32 | -634.61 | -1091.23 | -687.47 | -1006.31 | -633.98 |
| vpacx | -12838.65 | -9214.19 | -277.31 | -199.03 | 0.00 | 0.00 |

The global minimum portfolio values, compared to those for individual assets, are quite low.



*(red dot is tangency portfolio, orange dot is global min, names indicate approximate location)*

## 6. Asset Allocation

**Efficient Portfolio with No Short Sales Metrics and Monthly Target Return of .5%**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Asset** | **VaR.01** | **VaR.05** | **EF.Port.Weights** | **St.Dev** |
| **vfinx** | 7454.774553 | -177.446974 | 7.72% | 0.01717699 |
| **veurx** | 0 | 0 | 0 | Expected Return |
| **veiex** | 0 | 0 | 0 | 0.5% |
| **vbltx** | 42961.51812 | -1022.618636 | 44.49% |  |
| **vbisx** | 46148.14454 | -1098.470322 | 47.79% |  |
| **vpacx** | 0 | 0 | 0 |  |

**Tangency and T-Bills Portfolio**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Tangency and T-Bills Portfolio with No Short Sales and Monthly Target Return of 0.5% | | | | |
| Asset | VaR.01 | VaR.05 | EF.Port.Weights | StDev |
| vfinx | -51.32997049 | -37.77924347 | 1.84% | 0.01104972 |
| veurx | 0 | 0 | 0 | Expected Return |
| veiex | 0 | 0 | 0 | 0.50% |
| vbltx | 0 | 0 | 0.00% | T-Bills Allocation |
| vbisx | -5528.014648 | -4068.660265 | 198.64% | -1.004824 |
| vpacx | 0 | 0 | 0 | -$100482.4 |

The values at risk of these portfolios are much smaller than the other VaRs calculated above. The risk is quite diminished.

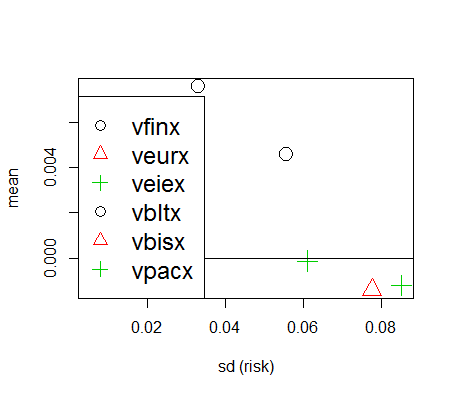
## 7. Single Index Model and CAPM

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Asset | Beta | Alpha | Residual SE | Multiple R^2 |
| VEURX | 1.290219 | -0.007304 | 0.03016 | 0.8515 |
| SE | 0.003908 | 0.070761 |  |  |
| VEIEX | 1.333994 | -0.007303 | 0.04221 | 0.7578 |
| SE | 0.005468 | 0.099013 |  |  |
| VBLTX | 0.00193 | 0.007553 | 0.03225 | 0.00001056 |
| SE | 0.004307 | 0.077995 |  |  |
| VBISX | 0.0087583 | 0.0026454 | 0.005519 | 0.007829 |
| SE | 0.0007149 | 0.0129462 |  |  |
| VPACX | 0.965625 | -0.004549 | 0.02945 | 0.7711 |
| SE | 0.003814 | 0.069075 |  |  |

## It appears that VEURX, VEIEX, and VPACX are all very market responsive, according to the Betas, while they are also responsible for most the market share. The estimates are not exactly that precise, however, averaging nearly 10% off. The bonds are least responsive and don’t have much market share at all. None of this is surprising; stocks make up more of the market usually.

|  |  |  |  |
| --- | --- | --- | --- |
| **Asset** | **Confidence Intervals** | **Intercept** | **VFINX** |
| VEURX | 2.5% | -0.01513 | 1.14858 |
|  | 97.5% | 0.00052 | 1.43186 |
| VEIEX | 2.5% | -0.01825 | 1.13580 |
|  | 97.5% | 0.00364 | 1.53219 |
| VBLTX | 2.5% | -0.00107 | -0.15419 |
|  | 97.5% | 0.01617 | 0.15806 |
| VBISX | 2.5% | 0.00121 | -0.01716 |
|  | 97.5% | 0.00408 | 0.03467 |
| VPACX | 2.5% | -0.01218 | 0.82736 |
|  | 97.5% | 0.00309 | 1.10389 |

Simply by using the confidence intervals above, the hypothesis that Beta is = 1 can be rejected at the 95% level for nearly every fund except VPACX.



There’d appear to be no linear relationship between the means and the standard deviations, however, a least squares regression could produce a linear result, possibly.

|  |  |  |  |
| --- | --- | --- | --- |
| Global Min. Variance Beta |  |  |  |
| **Asset** | **Weight** | **Beta** | **Beta.port** |
| vfinx | 5.63% | 1 | 0.0563 |
| veurx | -4.67% | 1.290219 | -0.060253227 |
| veiex | -1.53% | 1.333994 | -0.020410108 |
| vbltx | -9.92% | 0.00193 | -0.000191456 |
| vbisx | 108.33% | 0.0087583 | 0.009487866 |
| vpacx | 2.16% | 0.965625 | 0.0208575 |
|  |  | **Portfolio.Beta** | 0.000965096 |
| **Tangency Portfolio Beta** |  |  |  |
| **Asset** | **Weight** | **Beta** | **Beta.port** |
| vfinx | 0.17% | 1 | 0.001707 |
| veurx | -0.09% | 1.290219 | -0.001208935 |
| veiex | -0.02% | 1.333994 | -0.000297481 |
| vbltx | -0.02% | 0.00193 | -4.4004E-07 |
| vbisx | 0.99% | 0.0087583 | 8.63393E-05 |
| vpacx | -0.02% | 0.965625 | -0.000170916 |
|  |  | **Portfolio.Beta** | 1.92613E-05 |

As we can see from above, the portfolios minimized the beta responsiveness to the market; even though VFINX still was responsive, particularly in the minimum variance portfolio, overall the betas were quite diminished.